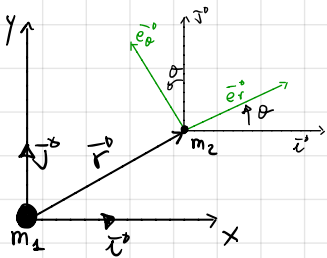


# Spacecraft and missions: Solutions for problem sheet 1.

1.



$$\begin{aligned}\vec{e}_r^0 &= \cos\theta \vec{i}^0 + \sin\theta \vec{j}^0 \\ \vec{e}_\theta^0 &= -\sin\theta \vec{i}^0 + \cos\theta \vec{j}^0\end{aligned}$$

NOTE  $\frac{d\vec{i}^0}{dt} = \frac{d\vec{j}^0}{dt} = 0!!$   
 $\vec{i}^0, \vec{j}^0$  are constant unit vectors!

$$\begin{aligned}\frac{d\vec{e}_r^0}{dt} &= -\sin\theta \dot{\theta} \vec{i}^0 + \cos\theta \dot{\theta} \vec{j}^0 = \dot{\theta} \vec{e}_\theta^0 \\ \frac{d\vec{e}_\theta^0}{dt} &= -\cos\theta \dot{\theta} \vec{i}^0 - \sin\theta \dot{\theta} \vec{j}^0 = -\dot{\theta} \vec{e}_r^0\end{aligned}$$

2.  $\vec{r}^0 = r \vec{e}_r^0 \quad \vec{v}^0 = \frac{d\vec{r}^0}{dt} = \dot{r} \vec{e}_r^0 + r \frac{d\vec{e}_r^0}{dt} = \dot{r} \vec{e}_r^0 + r \dot{\theta} \vec{e}_\theta^0$

$$\vec{h}^0 = \vec{r}^0 \times \vec{v}^0 = r \vec{e}_r^0 \times (\dot{r} \vec{e}_r^0 + r \dot{\theta} \vec{e}_\theta^0) = r \vec{e}_r^0 \times \dot{r} \vec{e}_r^0 + r \vec{e}_r^0 \times r \dot{\theta} \vec{e}_\theta^0 = r^2 \dot{\theta} \vec{e}_k^0$$

$\vec{e}_k^0$  points outside the plane

$\vec{e}_k^0$  unit vector orthogonal to the plane

3. 
$$\begin{aligned}\ddot{\vec{r}}^0 &= \frac{d\vec{v}^0}{dt} = \frac{d(\dot{r} \vec{e}_r^0 + r \dot{\theta} \vec{e}_\theta^0)}{dt} = \ddot{r} \vec{e}_r^0 + \dot{r} \dot{\theta} \vec{e}_\theta^0 + \dot{r} \dot{\theta} \vec{e}_\theta^0 + r \ddot{\theta} \vec{e}_\theta^0 - r \dot{\theta}^2 \vec{e}_r^0 = \\ &= (\ddot{r} - r \dot{\theta}^2) \vec{e}_r^0 + (2\dot{r} \dot{\theta} + r \ddot{\theta}) \vec{e}_\theta^0 = (\ddot{r} - r \dot{\theta}^2) \vec{e}_r^0 + \frac{1}{r} \frac{d}{dt} \left( \frac{1}{2} r^2 \dot{\theta} \right)\end{aligned}$$

$$4. \frac{d\vec{p}}{dt} = \vec{F} \Rightarrow m\ddot{\vec{r}} = \vec{F} = -\frac{\mu m}{r^3} \vec{r}$$

$$m(\dot{r} - r^2\dot{\theta}) \vec{e}_r \cdot \vec{e}_r = -\frac{\mu m}{r^3} \overbrace{\vec{r} \cdot \vec{e}_r}^r \Rightarrow m(\dot{r} - r^2\dot{\theta}) = -\frac{\mu m}{r^2}$$

$$m \frac{1}{r} \frac{d}{dt} \left( \frac{1}{2} r^2 \dot{\theta} \right) \vec{e}_\theta \cdot \vec{e}_\theta = -\frac{\mu m}{r^3} \underbrace{\vec{r} \cdot \vec{e}_\theta}_0 \Rightarrow \frac{1}{r} \frac{d}{dt} (r^2 \dot{\theta}) = 0 \Rightarrow r^2 \dot{\theta} = h = \text{const}$$

$\vec{r} \perp \vec{e}_\theta$